Platform design and product quality*

Berno Buechel,[‡†] Enrico Maria Fenoaltea,[§]
& Giovanni Valvassori Bolgè[‡]

[‡]University of Fribourg, Switzerland [§]Enrico Fermi Center (CREF), Italy

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Abstract

Product quality is crucial for consumer surplus and welfare. When firms sell on a platform, their tradeoff between costs and benefits of quality is affected. A platform not only defines the fee structure, but also which products are presented more prominently than others. Do these features induce the right incentives for product quality? We introduce a simple model to address this question. Firms decide on their product quality and price, while the platform determines how these choices map into the probability of making the customers aware of the product. Anticipating the symmetric Nash equilibrium among firms, the profit-maximizing platform chooses a transaction fee, a value fee, and the exponent parameter in a Tullock-like algorithm that rewards product quality. A transaction fee, or per-unit fee, would not distort the quality choices, but the platform does not use this instrument. It prefers to reward quality, which creates a contest for consumer attention that distorts quality upwards, and to simultaneously charge a value fee, which distorts product quality downwards. Both settings undermine welfare and result in net under-provision of product quality. Robustness of these insights is discussed in several model extensions.

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JEL Classification Codes: D47, D83, L81

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†Email address: `berno.buechel@unifr.ch`, web: `www.berno.info`.

1 Introduction

Many modern markets are organized with a central intermediary between sellers and buyers, e.g., online platforms. On such platforms there is a vast amount of different products. For instance, there are currently about two million apps in the AppStore and similarly in the Android Playstore. It is estimated there are more than 200 million different products on Alibaba.com and over 350 milion active products listed on Amazon.com. Considering a customer, be it a private person or a small company, it is evident that they cannot be aware of them all. As a consequence, firms compete for consumers' awareness of their products. Since the platform defines the presentation of products, it has a crucial role setting the rules of this competition for customers' limited attention. Many contributions have acknowledged this role of intermediaries and investigated how incentives for pricing, advertising, or platform commissions, set by the intermediary affect the market outcomes (e.g., Armstrong and Zhou, 2011, 2022; Athey and Ellison, 2011; Chen and He, 2011; Dinerstein et al., 2018; Eliaz and Spiegler, 2011; Hagiu and Jullien, 2011; Heidhues et al., 2023; Teh and Wright, 2022). However, as the vast majority of approaches have modeled product quality as fixed, they cannot be used to investigate incentives for product quality.

Importantly, by defining how product characteristics map into their presentation, the platform also sets incentives for firms' production decisions. In particular, a platform may present high-quality products more prominently than low-quality products, yielding additional incentives for firms to invest in product quality. Likewise, the fee structure defined by the platform may affect incentives of firms to produce at higher or lower quality. Since product quality is crucial for consumer surplus and welfare, it is essential to understand how the rules defined by the platform affect the quality choices of the firms. Thus, an important question arise: *Does the platform design—in terms of algorithm for prominence and the fee structure—induce the right incentives for firms' product quality?*

In this paper, we introduce a simple model of platform design that allows us to address this question. In the model firms decide on their product quality and price, while the platform determines how these choices map into the probability of presenting the product to the customers. In particular, the platform chooses a transaction fee (that is paid per unit sold), a value fee (that is paid proportional to the price at which a product is sold), and a quality-reward parameter (which is the exponent in a Tullock-like contest success function). The quality-reward parameter can be set positively, negatively or neutrally to make high quality products more likely, less likely, or equally likely to be considered by the customers. Negative rewards for quality would be an indirect way to incentivize lower prices. The benchmark level of quality is the welfare maximizing. It is characterized by the condition that marginal benefits of quality equal marginal cots of quality. A transaction fee leads to a form of the double marginalization problem but does not distort quality.[1] A value fee distorts quality downwards because firms bear all the costs of quality, while part of the benefits is captured by the platform. Both value fees and transaction fees are very common in practice.[2] Finally, the distortion by the quality-reward parameter depends on its sign. We solve the model for the profit-maximizing parameter choices by the platform (which anticipates that firms will play the unique symmetric Nash equilibrium). It turns out that the platform prefers not to charge any transaction fee, chooses a positive quality reward which distorts quality upwards, and charges a value fee that distorts quality downwards. In equilibrium firms under-provide product quality compared to the first best. Lowering the value fee and lowering the quality-reward parameter would both improve welfare. These results are unaltered in an extended model where the platform algorithm can condition not only on quality, but also on prices. We finally study some variations of the setup, in particular, concerning the demand and cost function and find our effects robust but accompanied by additional effects.

We contribute to the rather recent literature on intermediaries steering consumer demand. Based on theories of consumer search and of consideration sets (that we are not surveying here), consumers do not know all products or all product features. An intermediary guides the consumers' attention, which in turn, induces incentives for the firms that sell via this intermediary. While consumers value high match quality and low prices, the platform also considers the fees or commissions it can collect. Hence, a platform has to handle fundamental trade-offs between match quality for the consumers, intensity of (price) competition between the firms, and platform revenues based on fees (e.g., Armstrong and Zhou, 2011, 2022; Dinerstein et al., 2018; Heidhues et al., 2023; Nocke and Rey, 2024; Teh, 2022; Teh and Wright, 2022). Along numerous other findings, this literature has discovered effects of steering that are harmful for the consumers, including suboptimal matching between products and consumer preferences. However, by keeping product quality fixed, it does not analyze how platform design affects incentives to produce at higher or lower quality—our research question.

We found only two papers that directly address this question. First, Bergemann

[1]The transaction fee is set by a profit-maximizing monopolistic platform and the monopolistic firm adds a further markup. It distorts quantities like a quantity tax.

[2]Most platforms such as Amazon Market Place and Apple App Store charge substantial value fees, also known as "percentage cuts," in the range of 30%. This distorts quantity and price like a value tax. Theoretically, non-linear pricing would be efficient in terms of quantities.

and Bonatti (2024) study a theoretical framework where firms choose product quality for consumers on and off a platform. On the platform firms make personalized offers to consumers, which turn out to feature the socially efficient quality level (and efficient matching). Off the platform, quality is downwards distorted in several scenarios, which vary, e.g., by the information structure. Many effects are driven by the consumer's incentive-compatibility constraints, stemming from the choice between purchasing on and off the platform, which we do not address in our model. Another central difference is that firms pay a requested marketing budget to the platform for each product they present. This is unlike the value fee that distorts quality downwards in our model.

Second, Gao et al. (2023) use a complexity theory approach to study a platform that trades off higher fees against higher product quality. Firms either allocate a budget between fees and product quality, or choose the two freely. The platform maximizes profits by rewarding fees in the steering algorithm—which comes at the expense of product quality. Our model differs in some essential dimensions. First, in our model demand is based on microeconomics not on complexity theory. Second, we innovate by endogenizing the exponent in a Tullock contest success function, while in Gao et al. (2023)'s paper the exponent is fixed to one.[3] Finally, prices are fixed in that paper, while we let prices be chosen by the sellers. For fixed prices, it is less surprising that higher quality would improve welfare.

2 Model

2.1 Setup

Each firm $i \in N = \{1, 2, \dots, n\}$ chooses a quality $Q_i \geq 0$ and a price $p_i \geq 0$ for its product. The marginal costs of production $C(Q_i)$ are increasing and weakly convex in quality; i.e., $C'(Q_i) > 0$ and $C''(Q_i) \geq 0$.

A unit mass of consumers considers each product i with probability $A(Q_i, Q_{-i})$ (that results from some algorithm) and then demands $D(p_i, Q_i) = B(Q_i) - p_i$ of it. The benefit function $B(Q_i)$ is assumed to be increasing and concave in quality; i.e., $B'(Q_i) > 0$ and $B''(Q_i) < 0$.

A (monopolistic) platform chooses an algorithm that can potentially reward or dis-incentivize quality choices by making high quality products more or less prominent. In particular, it chooses parameter $\rho \in \mathbb{R}$ in the following Tullock-like function:

[3] Both models, in their general versions, consider a linear combination of two Tullock functions, where the relative weight is chosen by the platform. In our model, the platform also chooses the exponent, an aspect that has only been studied in a different context (e.g., by Sahm, 2022).

$A(Q_i, Q_{-i}) = \frac{Q_i^\rho}{\sum_{k \in N} Q_k^\rho}$, the algorithm. A literal interpretation of the model is that $A(Q_i, Q_{-i})$ is the share of consumers who are made aware of product i and that they only consider this product.[4]

Moreover, the platform defines the fee structure by setting a value fee $\theta \geq 0$ and a per-unit fee, called transaction fee $\tau \geq 0$. Hence, platform profits are $\Pi^{Pl} = \sum_i A(Q_i, Q_{-i}) \cdot [\theta p_i + \tau] \cdot D(p_i, Q_i)$. While firm i's profits are given by $\pi_i = A(Q_i, Q_{-i}) \cdot D(p_i, Q_i) \cdot [p_i(1-\theta) - C(Q_i) - \tau]$.

The timing of the model is as follows:

1. The platform chooses the algorithm's quality-reward parameter ρ, the value fee θ and the transaction fee τ.
2. The n firms choose their quality Q_i and price p_i (respectively, quantity y_i).
3. Consumers consider products according to the algorithm $A(Q_i, Q_{-i})$ and choose how much to buy according to their demand function $D(p_i, Q_i)$.

In Stage 1, the platform sets the parameters. In Stage 2, the firms play a simultaneous game. In stage 3, consumer behavior generates the payoffs for the firms and the platform.

2.2 Benchmark: First best

Overall welfare is the weighted sum of welfare in each market, while the weight for market i is given by $A(Q_i, Q_{-i})$. In each market, welfare consists of consumer surplus, producer surplus and platform surplus. The producer and platform surpluses are simply their profits. Consumer surplus in a market with price p_i, corresponding quantity y_i, and quality Q_i is $CS = \int_0^{y_i} (P(q, Q_i) - p_i) dq$, where $P(y, Q) := B(Q) - y$ is the inverse demand function. (In the inverse demand formulation it becomes apparent that the benefit function $B(Q)$ can be interpreted as the willingness-to-pay.)

Consider a social planner that can impose all decisions made by the platform and the firms. In every market, it will impose zero fees, price equal to marginal cost, and the optimal quality $\hat{Q} := \arg\max_Q \int_0^y P(q, Q) dq - C(Q)y$. Due to our assumptions on concavity and convexity, this maximization problem over quality has a unique solution, which is characterized by the first order condition $B'(\hat{Q}) = C'(\hat{Q})$. The algorithm that allocates attention with quality-reward parameter ρ then does not matter, as any market generates the same first best welfare.

[4]More general interpretations are discussed in Section 4.4 below.

Remark 1 (First best)**.** *The socially optimal choices of the platform and the firms satisfy:* $\hat{\theta} = \hat{\tau} = 0$*,* $\hat{p} = C(\hat{Q})$ *(and hence* $\hat{y} = B(\hat{Q}) - C(\hat{Q})$*) and*

$$B'(\hat{Q}) = C'(\hat{Q}). \tag{1}$$

Zero fees and prices equal to marginal costs assure that there is no welfare loss due to pricing. More importantly, the social planner would choose quality at its optimal level, where the marginal benefit of quality, which is the average increase in willingness-to-pay due to higher quality, equals the marginal costs of quality, as expressed by (1). This level of quality, $\hat{Q}$, will serve as a benchmark in the equilibrium analysis. In particular, we will call quality under-provided (over-provided) if it is below (above) $\hat{Q}$.

2.3 Firm's Problem

Each firm i solves the following problem:

$$\max_{Q_i, p_i} \qquad \pi_i = A(Q_i, Q_{-i}) \cdot D(p_i, Q_i) \cdot [p_i(1-\theta) - C(Q_i) - \tau] \ .$$

Using the inverse demand instead, the problem gets:

$$\max_{Q_i, y_i} \qquad \pi_i = A(Q_i, Q_{-i}) \cdot y_i \cdot [P(y_i, Q_i)(1-\theta) - C(Q_i) - \tau] \ .$$

The interior best response of i to Q_{-i} satisfies two first order conditions:[5]

$$y_i = \frac{P(y_i, Q_i)(1-\theta) - C(Q_i) - \tau}{-P'(y_i, Q_i)(1-\theta)} \tag{2}$$

$$\frac{A'(Q_i, Q_{-i})}{A(Q_i, Q_{-i})} = \frac{C'(Q_i) - \frac{\partial P(y_i, Q_i)}{\partial Q_i}(1-\theta)}{P(y_i, Q_i)(1-\theta) - C(Q_i) - \tau} \tag{3}$$

The first equation has the standard interpretation that optimal monopolistic quantity decreases (respectively, price increases) with costs, with fees and the price elasticity of demand. Denote the profit per unit sold as $v(y_i, Q_i) := P(y_i, Q_i)(1 - \theta) - C(Q_i) - \tau$. Then the second equation reads

$$\frac{A'(Q_i, Q_{-i})}{A(Q_i, Q_{-i})} = \frac{-\frac{\partial v(y_i, Q_i)}{\partial Q_i}}{v(y_i, Q_i)}$$

and can be interpreted as follows. Firms choose quality such that the relative gain

[5] We use the convention that $f'(x_1, x_2, ...) := \frac{\partial f(x_1, x_2, ...)}{\partial x_1}$.

in awareness (LHS) equals the relative gain in per-unit profit (RHS). Notice, that both sides of the equation can be positive, zero or negative, depending on whether quality is rewarded, neutrally treated, or dis-incentivized.

2.4 Symmetric equilibrium

Consider the second stage, where the platform parameters ρ, θ, and τ (chosen in the first stage) are fixed. For parameter ρ weakly positive, we can show directly that there is a unique symmetric Nash equilibrium between firms. For ρ negative, we need an additional assumption.[6]

Define the value of quality by $V(Q) := B(Q)(1-\theta) - C(Q) - \tau$.[7] Denote its roots by $\underline{Q} < \overline{Q}$, its maximizer by $\tilde{Q}$, and its elasticity by $\varepsilon(Q) := V'(Q)\frac{Q}{V(Q)}$.

Assumption 1. *Let the quality elasticity of value $\varepsilon(Q)$ be large around $\underline{Q}$, i.e., $\lim_{Q\to\underline{Q}} \varepsilon(Q) >> 0$, and then decreasing, i.e., $\varepsilon'(Q) < 0$ for any $Q \in (\underline{Q}, \overline{Q})$.*

The assumption means that the relative change in value due to a change in quality is high for low quality and gets lower the higher the quality. It is in line with the idea that the benefit function $B(Q)$ is strongly concave.

Remark 2. *The value function $V(Q)$ is a hill-shaped concave function that reaches its maximum at $\tilde{Q} \in (\underline{Q}, \overline{Q})$. It is positive in the interval $(\underline{Q}, \overline{Q})$.[8] We observe that the elasticity $\varepsilon(Q)$ must be decreasing for any $Q \geq \tilde{Q}$, while Assumption 1 assures that this property also holds for smaller Q.*

The first result shows existence of a unique symmetric equilibrium in Stage 2.

Proposition 1. *For any platform parameters ρ of moderate magnitude, $\theta \in [0, 1)$, and $\tau < B(\tilde{Q}) - c(\tilde{Q})$, there is a unique symmetric Nash equilibrium in the game between firms. Its quality $Q_i = Q, \forall i$ satisfies:*

$$Q \cdot \frac{C'(Q) - B'(Q)(1-\theta)}{B(Q)(1-\theta) - C(Q) - \tau} = \frac{\rho(n-1)}{2n}. \tag{4}$$

The proof of Proposition 1, as any proof, is relegated to the appendix. It uses that the LHS of Equation (4) is strictly increasing in Q, which is partially due to Assumption 1, while the RHS is constant. Although the equilibrium quality is only implicitly given, the proposition still reveals several comparative-static effects.

[6] A negative quality-reward parameter is at least thinkable. It could be one way to keep prices low and hence quantities high.

[7] The value function $V(Q)$ is strongly related to the per-unit profit function $v(y, Q)$, as $V(Q) = v(y, Q) + (1-\theta)y$. In particular, we have $V'(Q) = \frac{\partial v(y,Q)}{\partial Q}$ for any y, Q.

[8] If it were always negative, i.e., $V(\tilde{Q}) < 0$, then firms could only make negative profits, as $0 > V(Q) = B(Q)(1-\theta) - C(Q) - \tau \geq [B(Q) - y](1-\theta) - C(Q) - \tau = v(y, Q)$ for any y, Q.

For $\rho \equiv 0$, the firms choose quality such that the LHS of (4) is zero. That is obtained at $\tilde{Q}$, which was defined as the maximizer of $V(Q)$ and hence satisfies $B'(\tilde{Q})(1-\theta) = C'(\tilde{Q})$. Therefore, by concavity of B and convexity of C, the equilibrium quality is decreasing in θ. Only for $\theta = 0$ (and still $\rho \equiv 0$), the first best quality $\hat{Q}$ is chosen (with $B'(\hat{Q}) = C'(\hat{Q})$).

For $\rho > 0$, quality is chosen higher than $\tilde{Q}$, as the LHS of (4) must be positive. This upward distortion countervails the effect of the value fee. Finally, for $\rho < 0$, the firms' quality choice is distorted downwards, beyond the effect of the value fee: $Q < \tilde{Q} < \hat{Q}$. Hence, in this case quality must be under-provided.

The intuition for the downwards distortion by the value fee is that investments into quality increase costs $C(Q)$ and benefits $B(Q)$, differently for different actors: Whereas the costs are fully borne by the firm, only part $(1-\theta)$ of the consumer benefits go to the firm, the remaining part θ is captured by the platform. The intuition for the upwards distortion by the algorithm parameter ρ is rather straightforward. Additional reward for quality, i.e., $\rho > 0$, creates a quality competition among firms for consumer attention. In the absence of a value fee, $\theta = 0$, this leads to an over-provision of quality ($Q > \hat{Q}$), similar to the inefficiency of a Tullock contest. Finally, consider the effect of the number of firms. Larger n leads to (slightly) larger Q, in contrast to standard Tullock contests, as we will discuss further below.

3 Main results

In the first stage of the model, the platform operator chooses a platform design to maximize profits, anticipating the symmetric Nash equilibrium that will be played among the firms. The platform profit is given by $\sum_i \frac{Q_i^\rho}{\sum_{k\in N} Q_k^\rho} \cdot [\theta P(y_i, Q_i) + \tau] y_i$ and the unique Nash equilibrium of Stage 2 is characterized in Proposition 1. Hence, the platform problem can be written as

$$\max_{\rho,\theta,\tau} \qquad \Pi_{Pl} = \left[\theta \frac{B(Q)(1-\theta)+C(Q)+\tau}{2(1-\theta)} + \tau\right] \frac{B(Q)(1-\theta)-C(Q)-\tau}{2(1-\theta)} \qquad (\mathcal{P})$$

$$\text{subject to} \qquad (4), \theta \geq 0, \tau \geq 0.$$

Note: The restrictions on the parameters $\theta \in [0,1)$, $\tau < B(\tilde{Q}) - c(\tilde{Q})$, and ρ moderate used in Proposition 1 will be endogenously satisfied. Otherwise, firms could not make positive profits, would opt out (technically, they could offer $y_i = 0$), and the platform's profit would be zero as well.

We are now ready to state our main result.

Theorem 1. *The unique solution to the platform's problem $\mathcal{P}$ satisfies*

$$\begin{cases} (i) \quad \rho^* > 0 \ , \\ (ii) \quad \theta^* = \sqrt{1 - \left[\frac{C'(Q)}{B'(Q)}\right]^2} \ , \\ (iii) \quad \tau^* = 0. \end{cases}$$

Theorem 1 characterizes the optimal choice by the platform. First, it chooses a strictly positive reward for quality $\rho^* > 0$, creating quality competition for consumer attention. We know from the discussion of Proposition 1 that this per se leads to an upwards distortion of firm quality. At the same time, however, it chooses a positive value fee $\theta^* > 0$ that generates platform revenue and distorts firm quality downwards. Finally, it refrains from choosing a positive transaction fee τ^*.

It turns out that the upward distortion by the competition for attention and the downward distortion by the value fee have a net negative effect. This can be seen, e.g., from the second part of Theorem 1, where the optimal value fee $\theta^* = \sqrt{1 - \left[\frac{C'(Q)}{B'(Q)}\right]^2} > 0$ requires $B'(Q) > C'(Q)$ and hence, $Q < \hat{Q}$.[9]

Corollary 1. *In equilibrium, quality is under-provided.*

The main results show that a platform sets up a quality competition for consumer awareness that drives up qualities, but takes away a part of the firm revenue by charging a value fee. Combined the two actions induce firms to under-provide quality.

To illustrate the results and to have a quantification of the effects, let us study a numerical example.

Example. *Let $C(Q) = Q$ and $B(Q) = 10\sqrt{Q}$ and, where needed, $n = 2$. We get the first best quality $\hat{Q} = 25$, as the benchmark. From Theorem 1 and Proposition 1 we get the following equilibrium values: $\rho^* = 2\sqrt{2}$, $\theta^* = \frac{1}{3}$, $\tau^* = 0$, and $Q^* = 22.\overline{2}$.*

To see how the fee structure and the reward for quality distort qualities, let us study multiple scenarios where at least two parameters are set to zero. If $\rho \equiv 0$, $\theta \equiv 0$, and $\tau \equiv 0$, we receive optimal quality $\hat{Q} = 25$ (both in the first best scenario with marginal cost pricing, $\hat{p} = 25$, and in the second best with monopolistic pricing $p = 37.5$.) If $\rho \equiv 0$, $\theta \equiv 0$, and $\tau \equiv 12.5$, then the equilibrium quality is $Q = 25$. The transaction fee does not distort quality. If $\rho \equiv 0$, $\theta \equiv \frac{1}{3}$ and $\tau \equiv 0$, then the equilibrium quality is $Q = 11.1$. The value fee distorts quality downwards. If

[9]Similarly, this could be seen from (A.8) in the proof of Theorem 1, whose term $\sqrt{B'(Q)^2 - C'(Q)^2}$ requires the same. The proof also provides the expression for ρ^*.

$\rho \equiv 2\sqrt{2}$, $\theta \equiv 0$ and $\tau \equiv 0$, then the resulting quality is $Q = 50$. A reward for quality in terms of consumer awareness distorts quality upwards.

The following table summarizes these scenarios and provides each group's surplus for each scenario. Clearly, for consumers the first best is by far the best scenario. Letting firms choose the price, but keeping platform fees at zero (second best), reduces welfare because of monopoly pricing. The transaction fee further reduces welfare by a form double marginalization problem (where the platform's fee and the firm's price exceed their marginal costs). Setting a value fee instead harms welfare less. The reward for quality alone drives quality beyond its ideal point. The equilibrium combination of the instruments leads to a quality of $Q^* = 22.2$, notably below the welfare-maximizing quality of $\hat{Q} = 25$.

	Quality Q	**Welfare**	**Platform profit**	**Consumer surplus**	**Firm profit**	**Price** p	**Quantity** y
first best: $\hat{\rho} \equiv 0$, $\hat{\theta} \equiv 0$, $\hat{\tau} \equiv 0$, $\hat{p} \equiv 25$	25.0	312.5	0	312.5	0	25	25
second best: $\hat{\rho} \equiv 0$, $\hat{\theta} \equiv 0$, $\hat{\tau} \equiv 0$	25.0	234.38	0	78.13	156.25	37.50	12.50
only transaction fee: $\rho \equiv 0$, $\theta \equiv 0$, $\tau \equiv 12.5$	25.0	136.72	78.13	19.53	39.06	43.75	6.25
only value fee: $\rho \equiv 0$, $\tau \equiv 0$, $\theta \equiv \frac{1}{3}$	11.1	150.46	69.44	34.72	46.30	25	8.33
only reward for quality: , $\rho \equiv 2\sqrt{2}$, $\theta \equiv 0$, $\tau \equiv 0$	50.0	160.85	0	53.62	107.23	60.36	10.36
equilibrium: $\rho^* = 2\sqrt{2}$, $\tau^* = 0$, $\theta^* = \frac{1}{3}$	22.2	148.19	92.59	23.83	31.77	40.24	6.90

Figure 1 illustrates how quality and surpluses respond to different quality-reward parameters ρ and different value fees θ, keeping the other parameters at their equilibrium value. Observe that both firms and consumers preferred the fee and the quality-reward parameter to be zero, but the platform chooses them higher. This choice is excessive in the sense that reducing the parameter ρ, or reducing the value fee θ, would increase overall welfare.

In Proposition 1 we require the quality-reward parameter to be in a moderate range. In the example, that is $\rho \in (-2, \infty)$. In contrast to standard Tullock contests, there is no issue of non-existence for large exponents. For $\rho \to \infty$, we get $Q \to 100(1-\theta)^2 = 44.\bar{4}$.

4 Extensions and Robustness

4.1 Two-dimensional competition for awareness

So far, the platform's algorithm could only condition on firm qualities Q_i. Now, consider a variation of the model where the algorithm is a function of both quantity

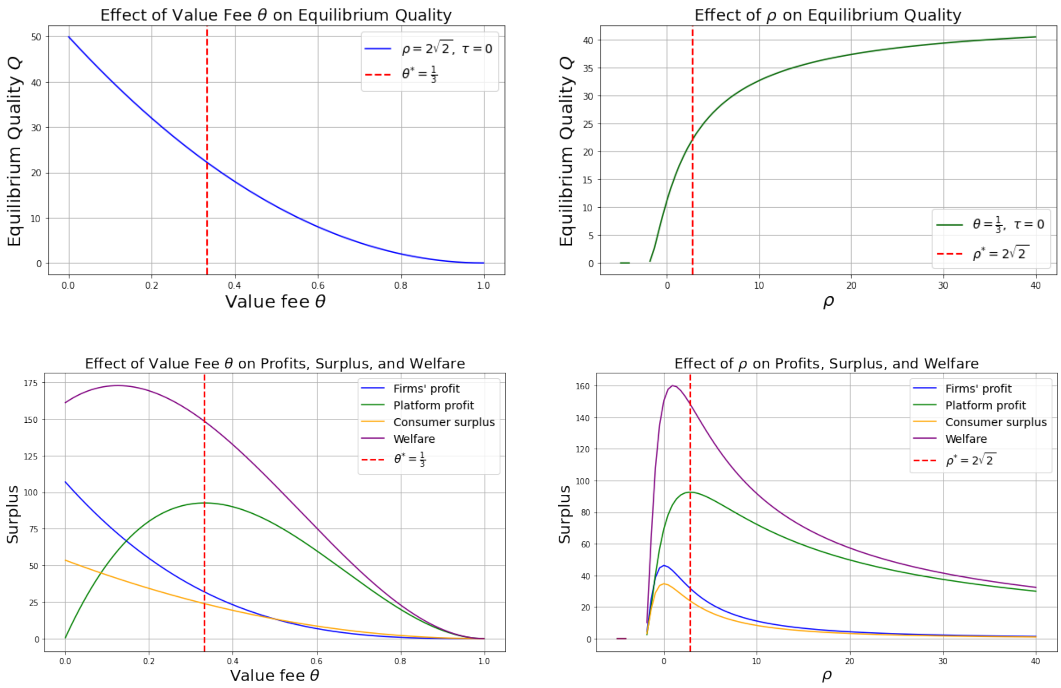

Figure 1: Numerical Example. Equilibrium quality and surpluses as a function of value fee θ and of quality reward parameter ρ.

and quality. We consider the following additive specification:

$$A(y_i, Q_i) = \alpha \frac{y_i^\sigma}{\sum_j^N y_j^\sigma} + (1-\alpha)\frac{Q_i^\rho}{\sum_j^N Q_j^\rho} \ , \tag{5}$$

with $\alpha \in [0,1]$ and $\sigma, \rho \geq 0$.

With the new parameter σ the platform can reward or punish quantity choices, which corresponds to punishing or rewarding prices. The new parameter α gives the platform the possibility to weigh the importance of quantity (respectively, prices) against the importance of quality for prominence. This model nests our model above when setting $\alpha \equiv 0$; and moves to a model with quantity (respectively, price) competition for awareness with increasing α. We formally define and solve this model in Appendix A.3. It turns out that the main result is robust to this extension.

Proposition 2. *For the extended platform problem, there exists a solution with*

$$\begin{cases} (i) \quad \rho^* > 0 \ , \\ (ii) \quad \theta^* = \sqrt{1 - \left[\frac{C'(Q)}{B'(Q)}\right]^2} \ , \\ (iii) \quad \alpha^* = \sigma^* = \tau^* = 0. \end{cases}$$

Proposition 2 shows that the platform does not use the option to reward quantities (respectively, punish high prices): $\alpha^* = \sigma^* = 0$. Thus, the equilibrium fee structure and the equilibrium choice of quality reward is chosen such as in the original model. While there might be other solutions, it turns out that the complementary platform choice, focusing on quantity and not on quality, is not among them. That is, in our model a platform does not find it optimal to ignore qualities and to reward or punish prices.

In both our baseline model and this extension we derive the platform-optimal algorithm for the given functional form. Finding the best algorithm in all generality is beyond the scope of this paper.[10] Note that the partial effects are not dependent on the functional form: Whenever a platform rewards quality, quality is distorted upwards, as it can be seen from Equation (3).

4.2 Quality investments

In our model, the costs of higher quality accrue for each unit that is produced. Indeed, the marginal cost of production $C(Q_i)$ are increasing in quality. Depending on the technology, there can additionally be bulk investments into quality, which improve the quality of each product. That would be a cost for production that is increasing in quality, say $K(Q)$, but fixed with respect to quantity. This changes a firm's profit function and its maximization problem becomes

$$\max_{Q_i, y_i} \qquad \tilde{\pi}_i = A(Q_i, Q_{-i}) \cdot y_i \cdot [P(y_i, Q_i)(1-\theta) - C(Q_i) - \tau] - K(Q_i) \ ,$$

where $C(Q_i)$ are the marginal costs of production and $K(Q_i)$ are the fixed costs.

The costs $K(Q_i)$ here are similar to the "effort" costs in a standard Tullock contest. The benefits however do not only capture the increase in probability of winning ($A(Q_i, Q_{-i})$), but also affect the "prize," as the inverse demand and the marginal costs depend on quality Q_i.

We focus again on symmetric equilibria and study the two first order conditions that led to Proposition 1. While the first (A.1) is unchanged (because fixed costs cancel out of the quantity optimization), the second (A.2) now becomes

$$\frac{\rho(n-1)}{Qn}\left[(B(Q) - y)(1-\theta) - C(Q) - \tau\right] = C'(Q) - B'(Q)(1-\theta) + nK'(Q). \quad (6)$$

[10] For a continuous set-up, the corresponding question is addressed in (Valvassori-Bolge, 2026).

Combining them yields

$$C'(Q) - B'(Q)(1-\theta) + nK'(Q) = \frac{\rho(n-1)}{Q \cdot 2n}\left[B(Q)(1-\theta) - C(Q) - \tau\right], \qquad (7)$$

which differs from (4) in Proposition 1 only by the term $nK'(Q)$.

For $\rho \equiv 0$, the firms choose quality such that the LHS of (7) is zero. That is obtained at a level of quality that is below the level $\tilde{Q}$, that would be chosen without fixed costs $K(Q)$. Intuitively, the additional costs of quality drive the equilibrium level of quality down. However, this is also true for the first best quality, which is now determined by $B'(\hat{Q}) - C'(\hat{Q}) = \frac{K'(\hat{Q})}{\hat{y}}$ and $\hat{y} = B(\hat{Q}) - C(\hat{Q})$. While the first best quality is constant in the number of firms, the symmetric equilibrium is not: the more firms n, the lower the chosen quality. This effect is known in standard contests, but was dominated in our baseline model, where costs of quality only have to be paid proportional to the share of awareness. Despite this difference, it turns out that the platform's choice given by Theorem 1 carries over to this extension.

4.3 More general demand functions

So far we have assumed quasi-linear demand. This choice is very convenient because it makes the change in demand due to quality changes independent of quantity. Formally, with quasi-linear demand we have $\frac{\partial P(y,Q)}{\partial Q} = B'(Q)$ and hence $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} = 0$. In general, this need not be the case. Spence (1975) shows that the cross-derivative effects lead to a downwards distortion of quality for $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} < 0$ and an upwards distortion for $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} > 0$. The monopolistic firm adjusts its quality choice to the marginal consumer, not the average consumer.

The Spence distortion comes on top of the two distortions that we investigate in this paper (customer awareness reward for quality and value fee). It does not change their substance. In particular, if the marginal consumer's willingness-to-pay for additional quality is low relative to the average consumer's, then quality is further lowered by optimizing firms.

4.4 More general consideration sets

A literal interpretation of our model was that each consumer considers only one product and their demand (given awareness) hence depends only on this product's price and quality. More realistically, most consumers consider a non-singleton subset of all products. Such an extension would not affect our analysis under the additional assumption that products are *totally differentiated*. (Clearly, a consumer who considers three totally differentiated products, chooses each product's quantity independently of the other products' characteristics.) Allowing for (partial) sustainability of different products and non-singleton consideration sets, generates firm competition on two levels. Firms compete for customers' limited attention to become part of their considerations sets; and they compete against the other products within any consideration sets.

This can be modeled following the framework of Dinerstein et al. (2018). Let $a_L(\mathbf{p}, \mathbf{Q})$ be the probability that a consumer faces consideration set $L \subseteq N$ for the profile of prices $\mathbf{p}$ and the profile of qualities $\mathbf{Q}$. Let $\tilde{D}_i(\mathbf{p}, \mathbf{Q}|L)$ be the demand for good i when a consumer considers the firms in L. Then expected demand is $\sum_{L\in 2^N} a_L(\mathbf{p}, \mathbf{Q}) \cdot \tilde{D}_i(\mathbf{p}, \mathbf{Q})|L)$. Our key assumption is that *expected demand is multiplicativly separable into probability of awareness and demand given awareness*: $\sum_{L\in 2^N} a_L(\mathbf{p}, \mathbf{Q}) \cdot \tilde{D}_i(\mathbf{p}, \mathbf{Q})|L) = A_i(\mathbf{p}, \mathbf{Q}) \cdot D_i(\mathbf{p}, \mathbf{Q})$ for an algorithm A_i and (average) demand function D_i.[11] [12] Hence, a re-interpretation of our demand function is that it covers *the average demand a firm faces.* The average is build over all possible considerations sets with averaging weights according to the probability that each set is obtained. In such a setting each firm chooses the best response to all other firms' choices and hence sets the quality according to the same incentives as discussed above. Indeed, stronger competition by closely substitutable products are mimicked by a highly elastic demand in our model, where firms act as monopolists. However, the model that we analyze is a simplified version in which all firms are the same. This is for tractability, as it allows us to use the symmetric solution in the game between firms in order to focus on the platform's problem.

5 Discussion

We develop a simple model to investigate how firms' quality choices depend on platform design. The profit-maximizing platform rewards quality by giving high quality products additional consumer attention. At the same time, it charges a value fee that distorts quality downwards such that quality is under-provided in equilibrium. Excessively high value fees—illustrated by a rate of 33 percent in our numerical example—may raise a social planner's concerns and serve as a starting point for potential policy remedies. Moreover, lowering the quality-reward parameter from its

[11] This separation is also possible in Dinerstein et al. (2018)'s framework. Their specification of demand for a product j, given by their equations (1)-(3), can be rewritten as $\sum_{L\in 2^L}(\prod_{i\in L} a_i)(\prod_{i\notin L}(1 - a_i))D_j(p_j, p_{-j}|L) = a_j \cdot \sum_{L\in 2^L: j\in L}(\prod_{i\in L\setminus j} a_i)(\prod_{i\notin L}(1 - a_i))D_j(p_j, p_{-j}|L)$. Hence, it is separable into a probability of considering product j, here called a_j and the remainder part, which can be interpreted as an average demand D_j.

[12] One implication of this independence assumption is that it does not allow platforms to produce compositions of products based on their substitutability. Maybe a platform would have incentives to present rather differentiated goods, an extreme case of which is treated by our model.

equilibrium value (while keeping all other parameters) would increase social welfare, albeit it pushed the resulting quality further below its optimal level. In that sense both the reward for quality and the value fee are set too high, maximizing platform profits at the expense of consumer and producer surplus.

We set up the model as simple as possible to then show the results' robustness in several extensions. However, there are other aspects that the model does not cover. First, in our model the platform does not discriminate between consumers. If platforms can target their offers to individual consumers, higher quality may increase the visibility of the product for some and decrease it for others at the same time. Hence, there are much richer options to set quality incentives. Second, we have assumed that the marginal costs of production $C(Q)$ are constant (they only depend on quality). If they also depend on the quantity, the problem gets substantially more complicated, as the absolute quantity provided also depends on the received awareness. Third, we have focused on homogeneous firms and symmetric equilibria to keep the model tractable. Of course, firm heterogeneity and specialization into different levels of quality are interesting aspects to study. Finally, we assumed away the problem of asymmetric information. As in our model platforms and consumers perfectly understand the quality of a considered product, the inefficiencies we uncover are not due to asymmetric information.

A Proofs

A.1 Proof of Proposition 1

Each Nash equilibrium must satisfy the two first order conditions Equations (2) and (3), which characterize the best response for each firm. In a symmetric equilibrium these conditions simplify to the equations below when using the quasi-linear demand and the functional form for the algorithm.

$$y = \frac{B(Q)(1-\theta) - C(Q) - \tau}{2(1-\theta)} \tag{A.1}$$

$$\frac{\rho(n-1)}{Qn} = \frac{C'(Q) - B'(Q)(1-\theta)}{[B(Q) - y](1-\theta) - C(Q) - \tau}. \tag{A.2}$$

Now, plugging in (A.1) into (A.2) yields

$$\rho\frac{n-1}{2n} = Q\frac{C'(Q) - B'(Q)(1-\theta)}{B(Q)(1-\theta) - C(Q) - \tau} \tag{A.3}$$

Using the definition of V given with Assumption 1, this reads

$$Q\frac{-V'(Q)}{V(Q)} = \rho\frac{n-1}{2n} \tag{A.4}$$

or

$$-\varepsilon(Q) = \rho\frac{n-1}{2n}. \tag{A.5}$$

Observe that the RHS is constant, while the LHS is continuously increasing by Assumption 1. It remains to be shown that for any value of ρ, there is a Q such that the equality holds.

First, suppose $\rho > 0$. At $Q = \tilde{Q}$, we have $-\varepsilon(Q) = 0 < \rho\frac{n-1}{2n}$. For $Q \to \bar{Q}$, we have $V(Q) \to 0$ while Q and $-V'(Q)$ are positive and bounded; hence $-\varepsilon(Q) \to \infty$ such that $-\varepsilon(Q) > \rho\frac{n-1}{2n}$. Therefore, there is a unique solution $Q^* \in (\tilde{Q}, \bar{Q})$ to Equation (A.5). Second, suppose $\rho = 0$. Then $Q^* = \tilde{Q}$ is the unique solution. Finally, suppose $\rho < 0$. By Assumption 1, we have $\lim_{Q \to \underline{Q}} \varepsilon(Q) >> 0$, which implies $-\varepsilon(Q) < \rho\frac{n-1}{2n} < 0$ for Q close enough to $\underline{Q}$. Since $-\varepsilon(\tilde{Q}) = 0$, we have a unique solution $Q^* \in (\underline{Q}, \tilde{Q})$ to Equation (A.5).

□

A.2 Proof of Theorem 1

It is straightforward to see that θ and τ cannot both be zero. Hence, without loss of generality we can write the Lagrangian of the problem $\mathcal{P}$ after taking logs as

$$\mathcal{L} = -2\ln(1-\theta) + \ln[\theta[B(Q)(1-\theta) + C(Q) - \theta] + 2\tau] + \ln[B(Q)(1-\theta) - C(Q) - \theta] + \nu_\theta \theta + \nu_\tau \tau$$

where ν_θ, ν_τ's are the non-negativity multipliers. The KKT conditions are as follows:

$$\frac{\theta\,[B'(Q)(1-\theta) + C'(Q)]}{\theta\,[B(Q)(1-\theta) + C(Q) - \tau] + 2\tau} - \frac{C'(Q) - B'(Q)(1-\theta)}{B(Q)(1-\theta) - C(Q) - \tau} = 0 \qquad [\rho]$$

$$\frac{2}{1-\theta} + \frac{B(Q)(1-\theta) + C(Q) - \tau - \theta B(Q)}{\theta\,[B(Q)(1-\theta) + C(Q) - \tau] + 2\tau} - \frac{B(Q)}{B(Q)(1-\theta) - C(Q) - \tau} + \nu_\theta = 0 \qquad [\theta]$$

$$\frac{2-\theta}{\theta\,[B(Q)(1-\theta) + C(Q) - \tau] + 2\tau} - \frac{1}{B(Q)(1-\theta) - C(Q) - \tau} + \nu_\tau = 0 \qquad [\tau]$$

$$\nu_\theta \geq 0 \qquad [\text{non-neg for } \theta]$$

$$\nu_\tau \geq 0 \qquad [\text{non-neg for } \tau]$$

$$\nu_\theta \theta = 0 \qquad [\text{CS for } \theta]$$

$$\nu_\tau \tau = 0 \qquad [\text{CS for } \tau].$$

Several cases need to be taken into account.

(i) Suppose that $\rho = 0$. It is implied by (A.2), when using $C'(Q) = B'(Q) * (1-\theta)$, that $[\rho]$ becomes

$$\frac{\theta\,[B'(Q)(1-\theta) + C'(Q)]}{\theta\,[B(Q)(1-\theta) + C(Q) - \tau] + 2\tau} = 0\,, \qquad \text{(A.6)}$$

which can hold if and only if $\theta = 0$. We now show that $\theta = 0$ leads to a contradiction. If $\theta = 0$, then $\nu_\theta > 0$ and $\nu_\tau = 0$. Then, $[\tau]$ becomes

$$\frac{1}{\tau} - \frac{1}{B(Q) - C(Q) - \tau} = 0$$
$$\tau = \frac{B(Q) - C(Q)}{2}\,.$$

We need to check that $[\theta]$ holds

$$2 + \frac{-B(Q) + C(Q) - \tau}{2\tau} + \nu_\theta = 0$$
$$2 + \frac{-B(Q) + C(Q) - \tau}{2\tau} \leq 0$$
$$3\tau - (B(Q) - C(Q)) \leq 0$$
$$B(Q) - C(Q) \leq 0\,,$$

which cannot hold. Therefore, θ cannot be zero. Consequently, (as $\rho = 0$ implies that $\theta = 0$, which is not possible), ρ cannot be zero.

(ii) Suppose $\rho < 0$. Using, (A.3), this implies $C'(Q) - B'(Q)(1-\theta) < 0$. This term appears in the condition $[\rho]$ above and implies that the second fraction must be negative. Thus, the condition $[\rho]$ can only hold if its first fraction is also negative. This is however impossible (given our assumptions $B'(Q) > 0$ and $C'(Q) > 0$). A contradiction. Hence, ρ must be strictly positive.

(iii) Suppose that $\nu_\theta = \nu_\tau = 0$. From $[\tau]$ we get:

$$\begin{aligned}(2-\theta)[B(Q)(1-\theta) - C(Q) - \tau] &= \theta[B(Q)(1-\theta) + C(Q) - \tau] + 2\tau \\ 2B(Q)(1-\theta)(1-\theta) - 2C(Q) &= 2\tau(2-\theta) \\ \tau &= \frac{B(Q)(1-\theta)^2 - C(Q)}{2-\theta} \ .\end{aligned}$$

Then, from $[\theta]$ we get

$$\begin{aligned}&2\theta[B(Q)(1-\theta) + C(Q) + \tau] + (1-\theta)[-B(Q)(1+\theta) + C(Q) - \tau] = 0 \\ &B(Q)(\theta - 1) + C(Q)(1+\theta) + \tau(3-\theta) = 0 \\ &\tau = \frac{B(Q)^2 - C(Q)(1+\theta)}{3-\theta} \ .\end{aligned}$$

Putting the two expressions for τ together yields

$$\begin{aligned}(3-\theta)[B(Q)(1-\theta)^2 - C(Q)] &= (2-\theta)[B(Q)(1-\theta)^2 - C(Q)(1+\theta)] \\ B(Q)(1-\theta)^2 &= C(Q)[3 - \theta - (2-\theta)(1+\theta)] \\ &= C(Q)(1 - 2\theta + \theta^2) \\ &= C(Q)(1-\theta)^2 \\ B(Q) &= C(Q) \ ,\end{aligned}$$

which cannot hold, otherwise profits would be negative. Hence, it must be the case that $\theta > 0$ and $\tau = 0$.

(iv) Thus, $\nu_\theta = 0$ and $\nu_\tau > 0$. Plugging $\tau = 0$ into $[\theta]$ yields

$$\begin{aligned}&\frac{2}{1-\theta} + \frac{B(Q)(1-\theta) + C(Q) - \theta B(Q)}{\theta[B(Q)(1-\theta) + C(Q)]} - \frac{B(Q)}{B(Q)(1-\theta) - C(Q)} = 0 \\ &[B(Q)(1-\theta) - C(Q)][B(Q) + C(1+\theta)] - B(Q)(1-\theta)\theta[B(Q)(1-\theta) + C(Q)] = 0 \\ &[B(Q)(1-\theta) + C(Q)][(1-\theta)B(Q)(1-\theta) - C(Q) - \theta B(Q)(1-\theta)] + \theta C(Q)[B(Q)(1-\theta) - C(Q)] \\ &[B(Q)]^2(1-\theta)^3 = [C(Q)]^2(1+\theta) \\ &\left[\frac{B(Q)}{C(Q)}\right]^2 = \frac{1+\theta}{(1-\theta)^3} \ .\end{aligned}$$

From $[\rho]$, we also get that

$$\begin{aligned}
\frac{B'(Q)(1-\theta)+C'(Q)}{B(Q)(1-\theta)+C(Q)} &= \frac{C'(Q)-B'(Q)(1-\theta)}{B(Q)(1-\theta)-C(Q)} \\
[B'(Q)(1-\theta)+C'(Q)][B(Q)(1-\theta)-C(Q)] &= [C'(Q)-B'(Q)(1-\theta)][B(Q)(1-\theta)+C(Q)] \\
B'(Q)B(Q)(1-\theta)^2 &= C'(Q)C(Q) \\
\frac{B(Q)}{C(Q)} &= \frac{C'(Q)}{B'(Q)}\frac{1}{(1-\theta)^2} \\
\left[\frac{B(Q)}{C(Q)}\right]^2 &= \left[\frac{C'(Q)}{B'(Q)}\right]^2 \frac{1}{(1-\theta)^4} \; .
\end{aligned}$$

Combining the two expressions yields

$$\theta^* = \sqrt{1-\left[\frac{C'(Q)}{B'(Q)}\right]^2} \; . \tag{A.7}$$

Finally, by substituting (A.7) and $\tau = 0$ into (4) and simplifying the numerator we receive

$$\rho^* = \frac{2n}{n-1}\cdot Q \cdot \frac{C'(Q)-B'(Q)+\sqrt{B'(Q)^2-C'(Q)^2)}}{B(Q)\left(1-\sqrt{1-\left[\frac{C'(Q)}{B'(Q)}\right]^2}\right)-C(Q)}. \tag{A.8}$$

□

A.3 Proof of Proposition 2

As in Section 2.3, the best responses of firm i are given by:

$$\begin{aligned}
\frac{\alpha\frac{\sigma\sum_{j\neq i} q_j^\sigma}{q_i\cdot(q_i^\sigma+\sum_{j\neq i} q_j^\sigma)}}{\alpha\frac{q_i^\sigma}{\sum_j^N q_j^\sigma}+(1-\alpha)\frac{Q_i^\rho}{\sum_j^N Q_j^\rho}} &= \frac{(-P')(1-\theta)}{P(q_i,Q_i)-C(Q_i)-\tau}-\frac{1}{q_i} \\
\frac{(1-\alpha)\frac{\rho\sum_{j\neq i} Q_j^\rho}{Q_i\cdot(Q_i^\rho+\sum_{j\neq i} Q_j^\rho)}}{\alpha\frac{q_i^\sigma}{\sum_j^N q_j^\sigma}+(1-\alpha)\frac{Q_i^\rho}{\sum_j^N Q_j^\rho}} &= \frac{C'(Q)-\frac{\partial P(q,Q)}{\partial Q}(1-\theta)}{P(q,Q)(1-\theta)-C(Q)-\tau} \; .
\end{aligned}$$

We focus on symmetric equilibria. After substituting the quasilinear utility specification and rearranging, the two equations can be rewritten as

$$q = \frac{B(Q)-C(Q)-\tau}{(1-\theta)}\left[\frac{1+\alpha\frac{\sigma(n-1)}{n}}{2+\alpha\frac{\sigma(n-1)}{n}}\right] \tag{A.9}$$

$$(1-\alpha)\frac{\rho(n-1)}{nQ} = \frac{C'(Q)-B'(Q)(1-\theta)}{(B(Q)-q)(1-\theta)-C(Q)-\tau} \; , \tag{A.10}$$

which yield

$$(1-\alpha)\frac{\rho(n-1)}{nQ} = \left[2+\alpha\frac{\sigma(n-1)}{n}\right]\frac{C'(Q)-B'(Q)(1-\theta)}{B(Q)(1-\theta)-C(Q)-\tau} \quad . \tag{A.11}$$

The platform solves the following problem:

$$\begin{aligned}\max_{\alpha,\sigma,\rho,\theta,\tau} \quad \Pi^{Pl} = & \frac{1}{(1-\theta)^2\left[2+\alpha\frac{\sigma(n-1)}{n}\right]^2}\left[\theta\left[B(Q)(1-\theta)+C\left[1+\alpha\frac{\sigma(n-1)}{n}\right]-\tau\right]\right]+ \\ & +\tau\left[2+\alpha\frac{\sigma(n-1)}{n}\right][B(Q)(1-\theta)-C(Q)-\tau]\left[1+\alpha\frac{\sigma(n-1)}{n}\right] \\ & \text{subject to} \qquad (A.11), \theta \geq 0, \tau \geq 0. \end{aligned} \tag{$\mathcal{P}'$}$$

As in the proof of Theorem 1, we take logs and then study the KKT conditions. There are now two extra conditions

$$\begin{aligned}
\frac{(-2)\frac{\sigma(n-1)}{n}}{2+\alpha\frac{\sigma(n-1)}{n}} + \frac{\theta C(Q)\frac{\sigma(n-1)}{n}+\tau\frac{\sigma(n-1)}{n}}{\theta\left[B(Q)(1-\theta)+C\left[1+\alpha\frac{\sigma(n-1)}{n}\right]-\tau\right]+\tau\left[2+\alpha\frac{\sigma(n-1)}{n}\right]} + \frac{\frac{\sigma(n-1)}{n}}{1+\alpha\frac{\sigma(n-1)}{n}} + [\rho] &= 0 \\
\frac{(-2)\frac{\alpha(n-1)}{n}}{2+\alpha\frac{\sigma(n-1)}{n}} + \frac{\theta C(Q)\frac{\alpha(n-1)}{n}+\tau\frac{\alpha(n-1)}{n}}{\theta\left[B(Q)(1-\theta)+C\left[1+\alpha\frac{\sigma(n-1)}{n}\right]-\tau\right]+\tau\left[2+\alpha\frac{\sigma(n-1)}{n}\right]} + \frac{\frac{\alpha(n-1)}{n}}{1+\alpha\frac{\sigma(n-1)}{n}} + [\rho] &= 0 \\
\frac{\theta\left[B'(Q)(1-\theta)+C'(Q)\right]}{\theta\left[B(Q)(1-\theta)+C-\tau\right]+2\tau} - \frac{C'(Q)-B'(Q)(1-\theta)}{B(Q)(1-\theta)-C-\tau} &= 0 \\
\frac{2}{1-\theta} + \frac{B(Q)(1-\theta)+C(Q)-\tau-\theta B(Q)}{\theta\left[B(Q)(1-\theta)+C-\tau\right]+2\tau} - \frac{B(Q)}{B(Q)(1-\theta)-C-\tau} + \nu_f &= 0 \\
\frac{2-\theta}{\theta\left[B(Q)(1-\theta)+C-\tau\right]+2\tau} - \frac{1}{B(Q)(1-\theta)-C-\tau} + \nu_\tau &= 0 \\
\nu_\theta &\geq 0 \\
\nu_\tau &\geq 0 \\
\nu_\theta\theta &= 0 \\
\nu_\tau\tau &= 0.
\end{aligned}$$

Clearly, when $\alpha = \sigma = 0$, the system of equations reduces to the same system as in the proof of Theorem 1. Moreover, the conditions for $[\alpha]$ and $[\sigma]$ are satisfied. □

References

Armstrong, M. and Zhou, J. (2011). Paying for prominence. *The Economic Journal*, 121(556):F368–F395.

Armstrong, M. and Zhou, J. (2022). Consumer information and the limits to competition. *American Economic Review*, 112(2):534–577.

Athey, S. and Ellison, G. (2011). Position auctions with consumer search. *The Quarterly Journal of Economics*, 126(3):1213–1270.

Bergemann, D. and Bonatti, A. (2024). Data, competition, and digital platforms. *American Economic Review*, 114(8):2553–2595.

Chen, Y. and He, C. (2011). Paid placement: Advertising and search on the internet. *The Economic Journal*, 121(556):F309–F328.

Dinerstein, M., Einav, L., Levin, J., and Sundaresan, N. (2018). Consumer price search and platform design in internet commerce. *American Economic Review*, 108(7):1820–59.

Eliaz, K. and Spiegler, R. (2011). Consideration sets and competitive marketing. *The Review of Economic Studies*, 78(1):235–262.

Gao, F., Fenoaltea, E. M., and Zhang, Y.-C. (2023). Market failure in a new model of platform design with partially informed consumers. *Physica A: Statistical Mechanics and its Applications*, 619:128748.

Hagiu, A. and Jullien, B. (2011). Why do intermediaries divert search? *The RAND Journal of Economics*, 42(2):337–362.

Heidhues, P., Köster, M., and Kőszegi, B. (2023). Steering fallible consumers. *The Economic Journal*, 133(652):1430–1465.

Nocke, V. and Rey, P. (2024). Consumer search, steering, and choice overload. *Journal of Political Economy*, 132(5):1684–1739.

Sahm, M. (2022). Optimal accuracy of unbiased tullock contests with two heterogeneous players. *Games*, 13(2):24.

Spence, A. M. (1975). Monopoly, quality, and regulation. *The Bell Journal of Economics*, pages 417–429.

Teh, T. H. (2022). Platform governance. *American Economic Journal: Microeconomics*, 14(3):213–254.

Teh, T. H. and Wright, J. (2022). Intermediation and steering: Competition in prices and commissions. *American Economic Journal: Microeconomics*, 14(2):281–321.

Valvassori-Bolge, G. (2026). Extreme points and large contests. Mimeo, University of Fribourg.